5 Polyphase induction machine

- 5.2 Construction and principle of operation
- Machine construction

Magnetic part

Stator and rotor windings

Electrical degrees

=pairs of poles× Mechanical degree

Induced Emf:

$$e_{as} = -\frac{d\lambda}{dt} = -\frac{d}{dt}\{(k_{w1}T_1)\Phi_m \sin(2\pi f_s t)\} = -2\pi f_s k_{w1} T_1 \Phi_m \cos(2\pi f_s)$$

The rotating magnetic field

- A simple three-phase stator

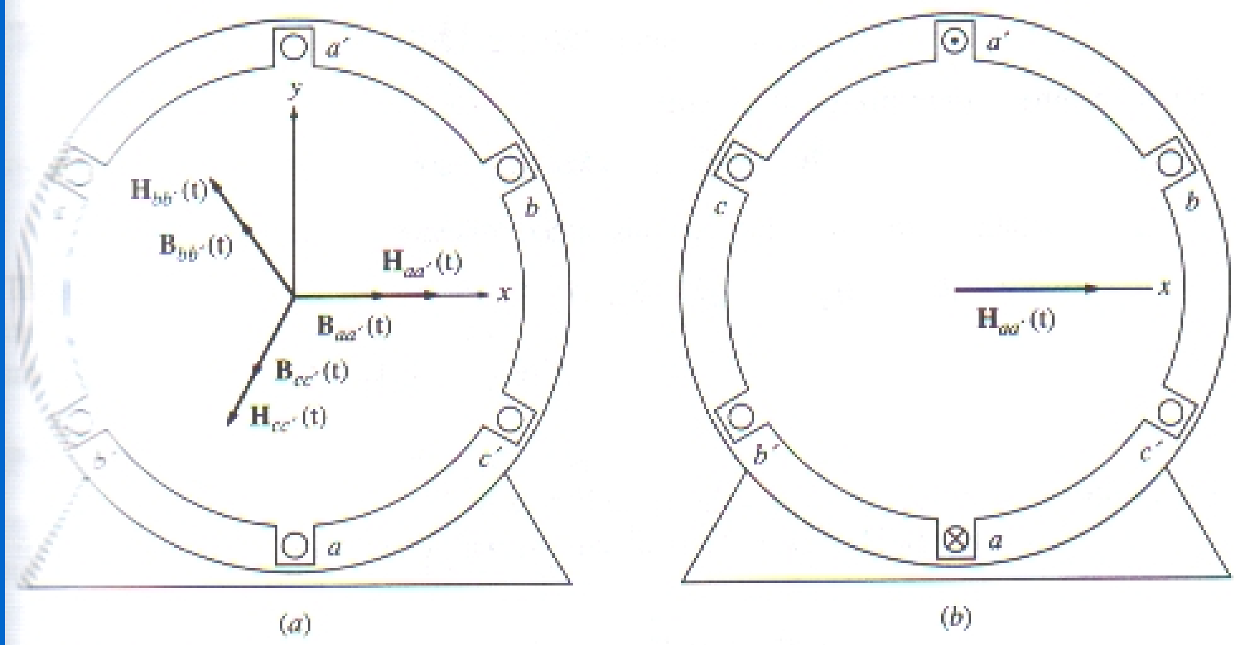

FIGURE 4–8
(a) A simple three-phase stator. Currents in this stator are assumed positive if they flow into the unprimed end and out the primed end of the coils. The magnetizing intensities produced by each coil are also shown. (b) The magnetizing intensity vector $\mathbf{H}_{aa'}(t)$ produced by a current flowing in coil aa'.

- Assume that the currents in three coils

$i_{aa'} = I_M \sin\omega t$

$i_{bb'} = I_M \sin(\omega t - 120^o)$

$i_{cc'} = I_M \sin(\omega t - 240^o)$

- Their magnetic field intensity

$H_{aa'} = H_M \sin\omega t \angle 0^o$

$H_{bb'} = H_M \sin(\omega t - 120^o) \angle 120^o$

$H_{cc'} = H_M \sin(\omega t - 240^o) \angle 240^o$

- Their magnetic flux densities

$$B_{aa'} = B_M \sin\omega t \angle 0^o$$

$$B_{bb'} = B_M \sin(\omega t - 120^o) \angle 120^o$$

$$B_{cc'} = B_M \sin(\omega t - 240^o) \angle 240^o$$

Note that $B = \mu H$.

- At time $\omega t = 0^o$

$$B_{aa'} = 0$$

$$B_{bb'} = B_M \sin(-120^o) \angle 120^o$$

$$B_{cc'} = B_M \sin(-240^o) \angle 240^o$$

- At time $\omega t = 90°$

$$B_{aa'} = B_M \sin 90° \angle 0°$$

$$B_{bb'} = B_M \sin(-30°) \angle 120°$$

$$B_{cc'} = B_M \sin(-150°) \angle 240°$$

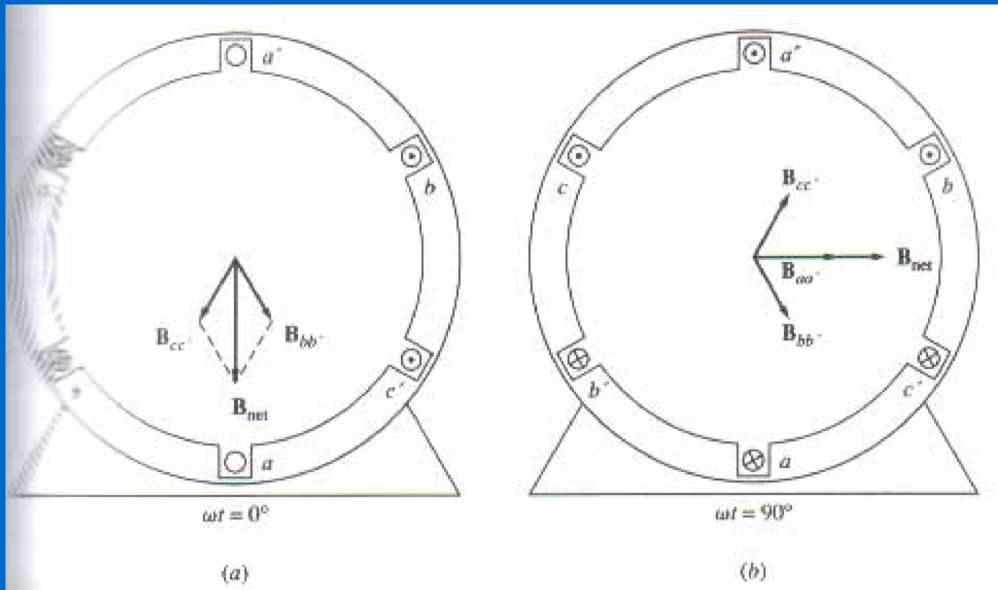

FIGURE 4–9
(a) The vector magnetic field in a stator at time ωt = 0°. (b) The vector magnetic field in a stator at time ωt = 90°.

Proof ...

- The net magnetic flux density

$B_{net}(t)= B_{aa'} + B_{bb'} + B_{cc'} = B_M\sin\omega t\angle 0^o+ B_M\sin(\omega t-120^o)\angle 120^o+B_M\sin(\omega t-240^o)\angle 240^o$

$= B_M\sin\omega t$

$-(1/2)B_M\sin(\omega t-120^o)+j(\sqrt{3}/2)B_M\sin(\omega t-120^o)$

$-(1/2)B_M\sin(\omega t-240^o)-j(\sqrt{3}/2)B_M\sin(\omega t-240^o)$

$B_{net}(t) = 1.5B_M\sin\omega t - j1.5B_M\sin\omega t$

Proof …

- Therefore

$$\left|B_{net}\right| = 1.5B_M$$

$$\angle B_{net} = \tan^{-1}\frac{-\cos\omega t}{\sin\omega t} = \omega t - 90°$$

- Principle of operation

Synchronous speed

$$\omega_s = 2\pi f_s, \quad \text{rad/sec}$$

If ω_m is the mechanical rotor speed, slip speed is

$$\omega_{s1} = \omega_s, - \omega_r = \omega_s - p\omega_m/2, \quad \text{rad/sec}$$

The differential speed between the stator magnetic field and rotor windings is slip speed which is defined as

$$s = \frac{\omega_{s1}}{\omega_s}$$

The rotating magnetic field

- A simple three-phase stator

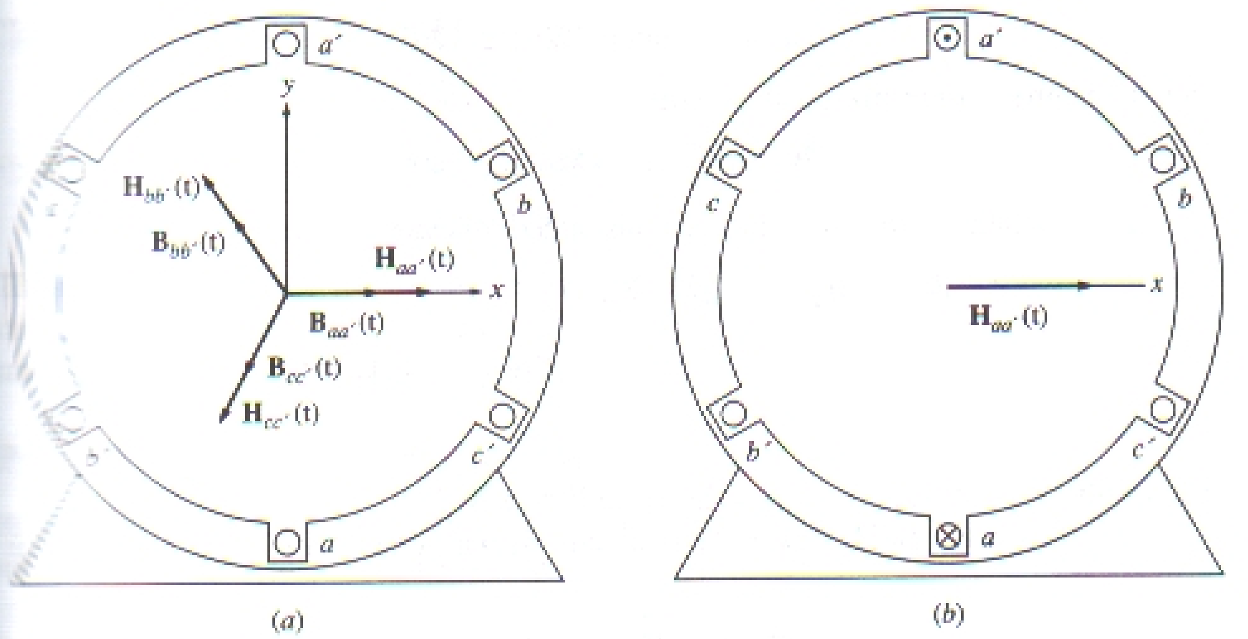

FIGURE 4–8

(a) A simple three-phase stator. Currents in this stator are assumed positive if they flow into the unprimed end and out the primed end of the coils. The magnetizing intensities produced by each coil are also shown. (b) The magnetizing intensity vector $H_{aa'}(t)$ produced by a current flowing in coil aa'.

The rotor electrical speed is

$$\omega_r = \omega_s(1 - s), \quad \text{rad/sec}$$

The rotor speed in rpm, denoted by n_r is

$$n_r = n_s(1 - s), \quad \text{(rpm)}$$

where $n_s = \frac{120 f_s}{p}$

5.3 Induction-motor equivalent circuit

- Fig. 5.1 Elementary equivalent circuit.

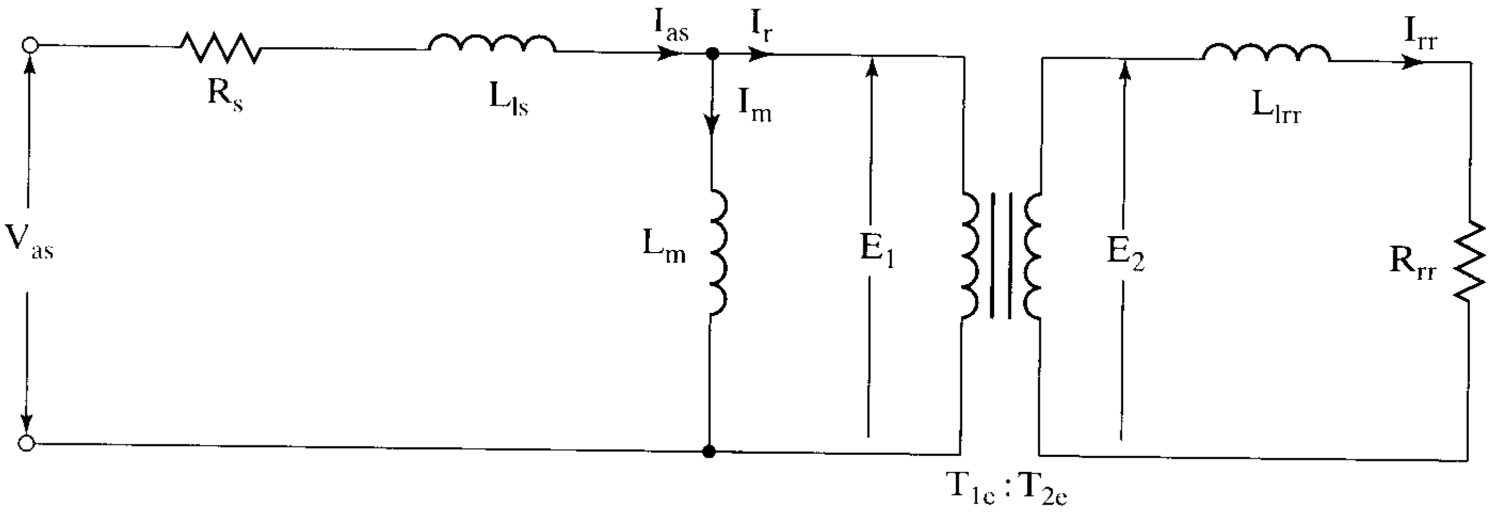

Figure 5.1 Elementary equivalent circuit for the induction motor

- The relationship between the induced emfs is

$$\frac{E_2}{E_1} = s\frac{T_{2e}}{T_{1e}} = \frac{s}{a}$$

where T1e and T2e are the effective stator and rotor turns per phase, and a is the turns ratio.

- The rotor current I_{rr}, then, is

$$I_{rr} = \frac{E_2}{R_{rr} + j\omega_{s1}L_{1rr}} = \frac{E_2}{R_{rr} + js\omega_s L_{1rr}}$$

- The rotor current is also (from stator)

$$I_{rr} = \frac{E_1}{\frac{aR_{rr}}{s} + j\omega_s a L_{1rr}} = \frac{E_1 / a}{\frac{R_{rr}}{s} + j\omega_s L_{1rr}}$$

- The rotor current reflected into the stator is denoted as I_r as

$$I_r = \frac{I_{rr}}{a}$$

$$I_r = \frac{E_1}{\frac{(a^2 R_{rr})}{s} + j\omega_s (a^2 L_{rr})} = \frac{E_1}{\frac{R_r}{s} + j\omega_s L_{1r}}$$

- Fig. 5.2 is equivalent circuit of the induction motor.

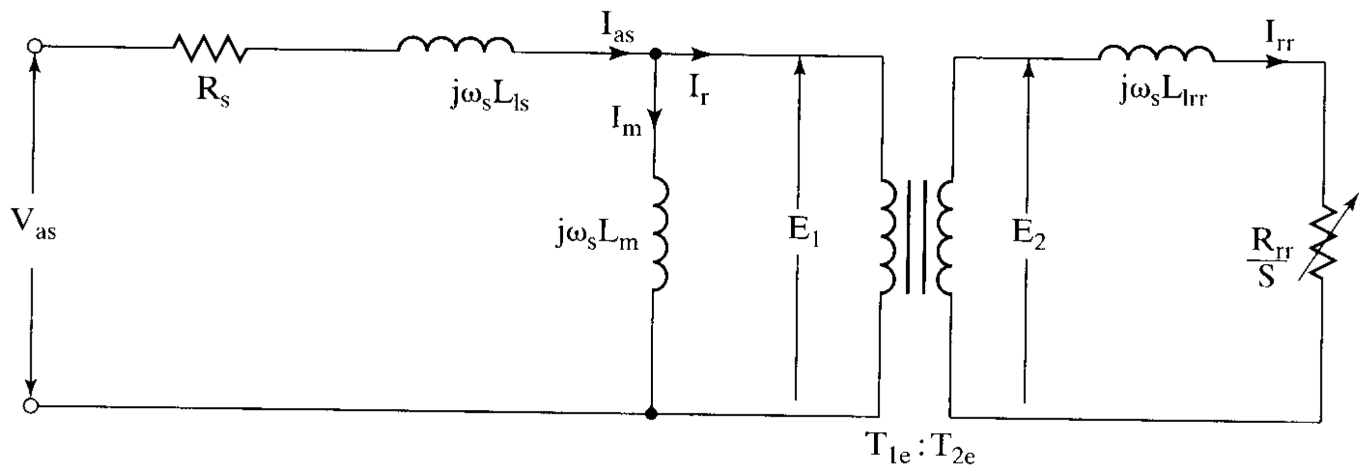

Figure 5.2 Equivalent circuit of the induction motor

• Fig. 5.3 Equivalent circuit with the rotor at stator frequency.

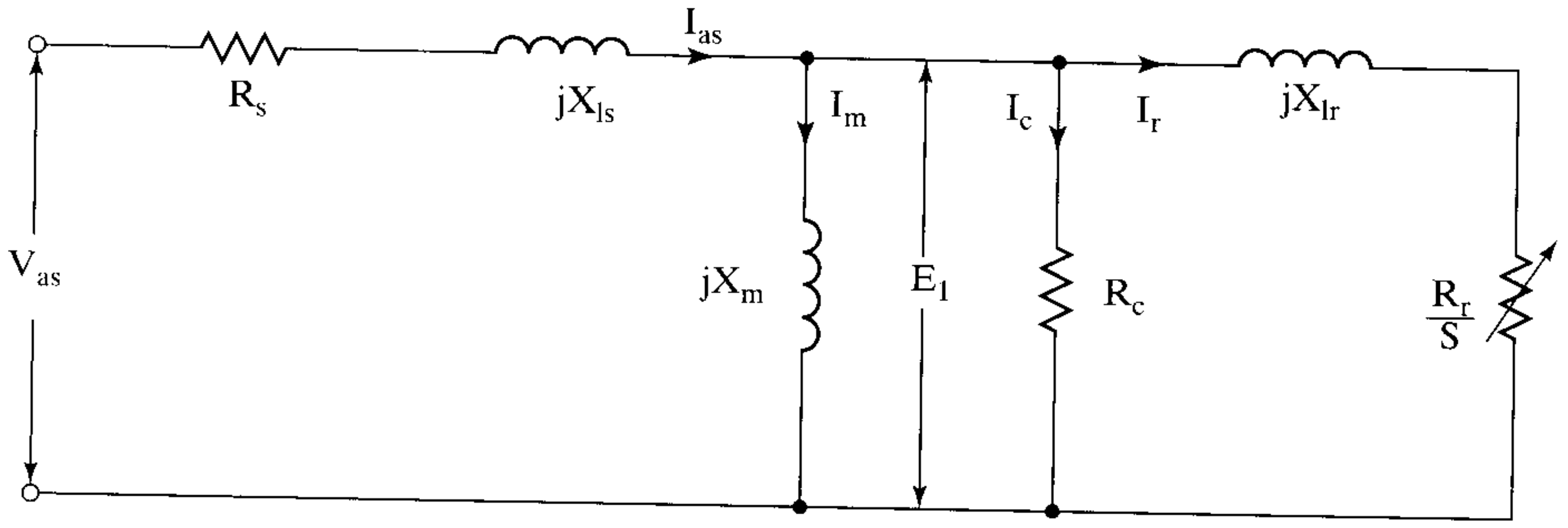

Figure 5.3 Equivalent circuit with the rotor at stator frequency

- The no-load current

$$I_o = I_m + I_c$$

- The core-loss component of the stator current is

$$I_c = \frac{E_1}{R_c}$$

- The rotor phase current is

$$I_r = \frac{E_1}{\frac{R_r}{s} + jX_{1r}}$$

- The stator current then is

$$I_{as} = I_r + I_o$$

- Fig. 5.4 Simplified phasor diagram of the induction motor.

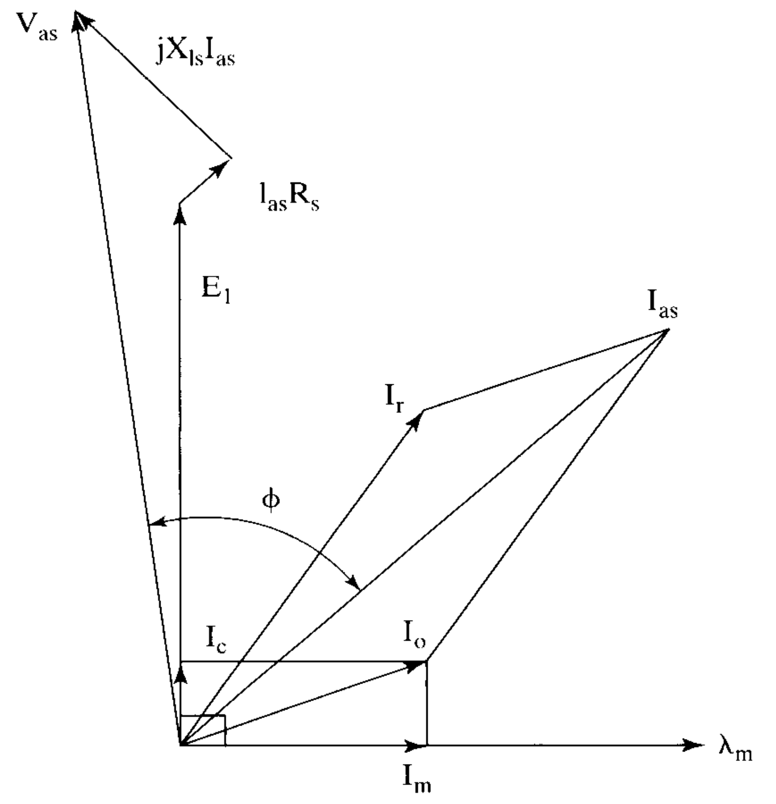

Figure 5.4 Simplified phasor diagram of the induction motor

- The applied stator input voltage

$$V_{as} = E_1 + (R_s + jX_{1s})I_{as}$$

- 5.4 Steady-state performance equations of the induction motor
- The air-gap power is

$$P_a = P_i - 3I_s^2 R_s$$

- Neglecting the core losses, the air-gap power is equal to the total power dissipated in R_r/s

$$P_a = 3I_r^2 \frac{R_r}{s} = 3I_r^2 R_r + 3I_r^2 R_r \frac{(1-s)}{s}$$

- The mechanical power output is

$$P_m = 3I_r^2 R_r \frac{(1-s)}{s}$$

- Alternately, the mechanical power output

$$P_m = T_e \omega_m$$

- Hence,

$$T_e = \frac{3I_r^2 R_r (1-s)}{s\omega_m}$$

- Let the rotor speed be in terms of the slip and stator frequency

$$\omega_m = \frac{\omega_r}{P/2} = \frac{\omega_s (1-s)}{P/2}$$

- The electromagnetic or air gap torque is

$$T_e = 3\left(\frac{P}{2}\right)\frac{I_r^2 R_r}{s\omega_s}$$

- The shaft output power of the machine is

$P_s = P_m - P_{fw}$

5.5 Steady-state performance

- Fig. 5.6 (a) Induction motor speed-torque characteristic.

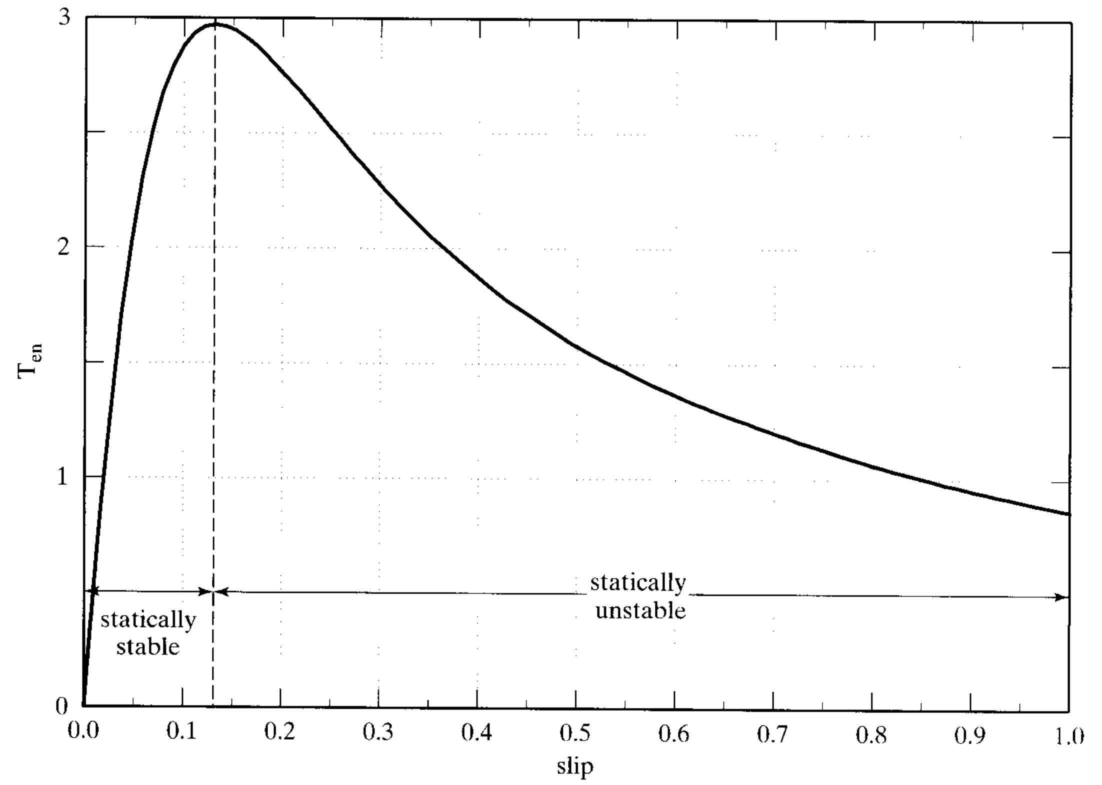

Figure 5.6 (a) Induction motor speed–torque characteristics

- Fig. 5.6 (b) Generation and braking characteristic of the induction motor.

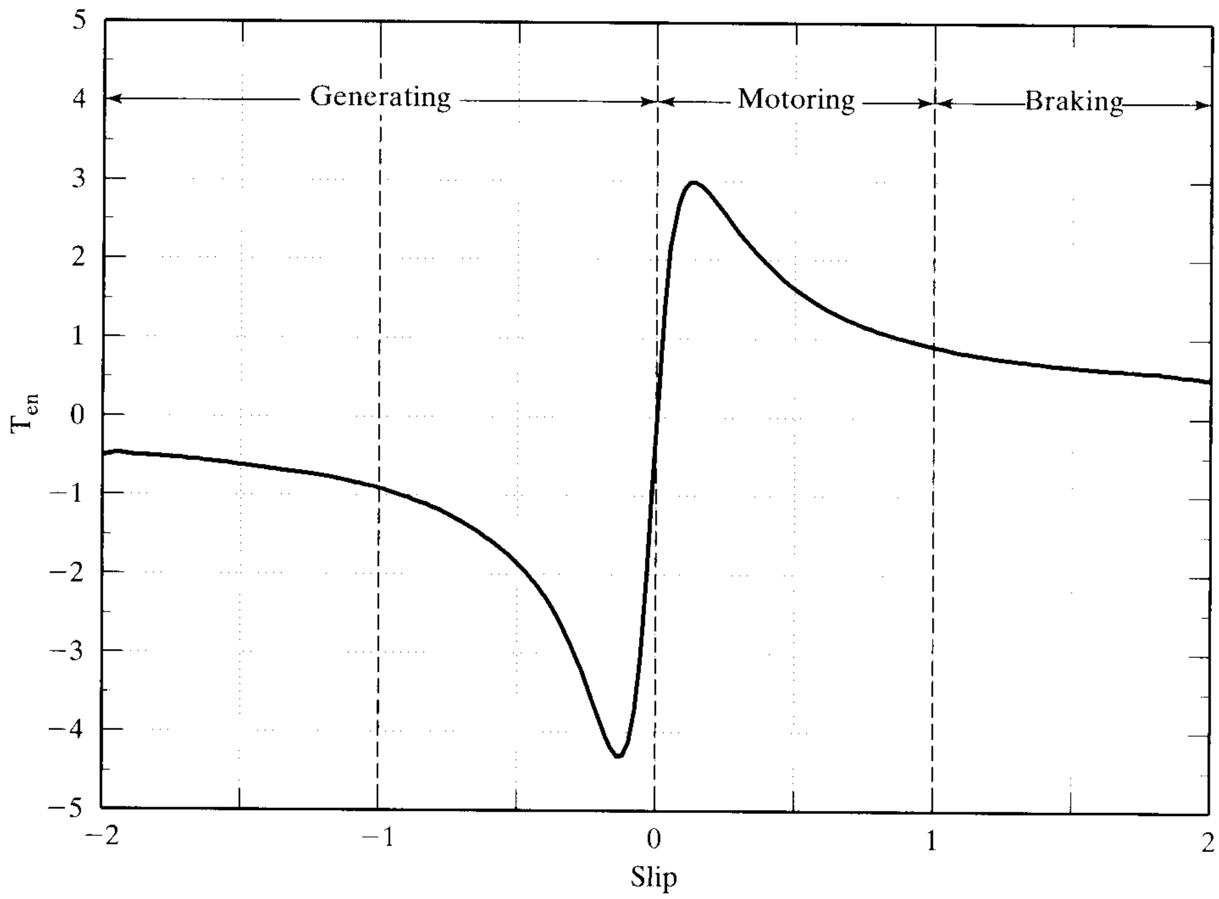

Figure 5.6 (b) Generation and braking characteristics of the induction motor

- Fig 5.7 Performance characteristic of an induction motor.

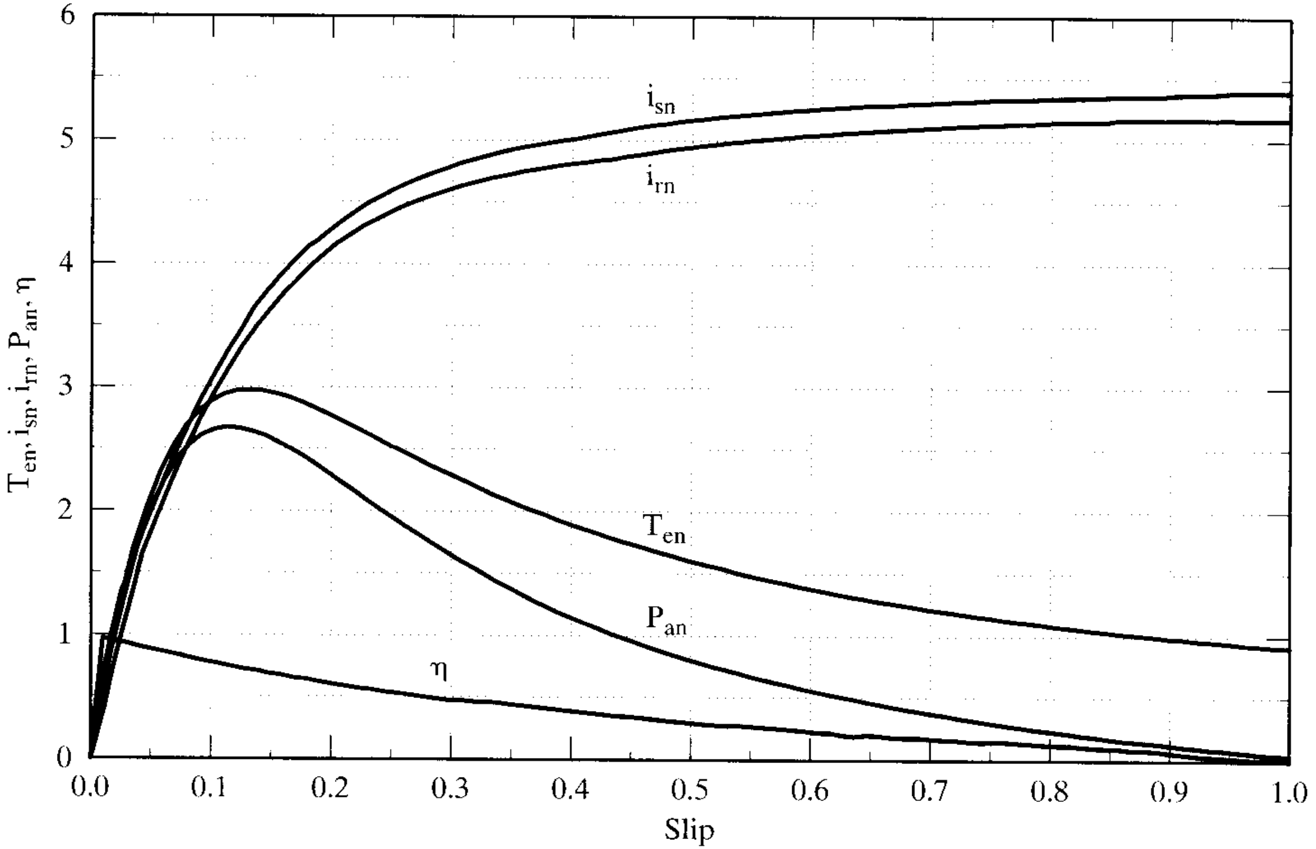

Figure 5.7 Performance characteristics of an induction motor

- Starting torque: (s = 1)

$$T_e \cong \frac{3}{\omega_s} \cdot \frac{P}{2} \cdot \frac{V_{as}^2 \cdot R_r}{(R_s + R_r)^2 + (X_{1s} + X_{1r})^2}$$

- 5.6 Measurement of motor parameters
- Stator resistance
- No-Load test

The no-load power factor

$$\cos\phi_0 = \frac{P_1}{V_{as} I_0}$$

Fig. 5.10 Equivalent circuit of the induction motor at no load.

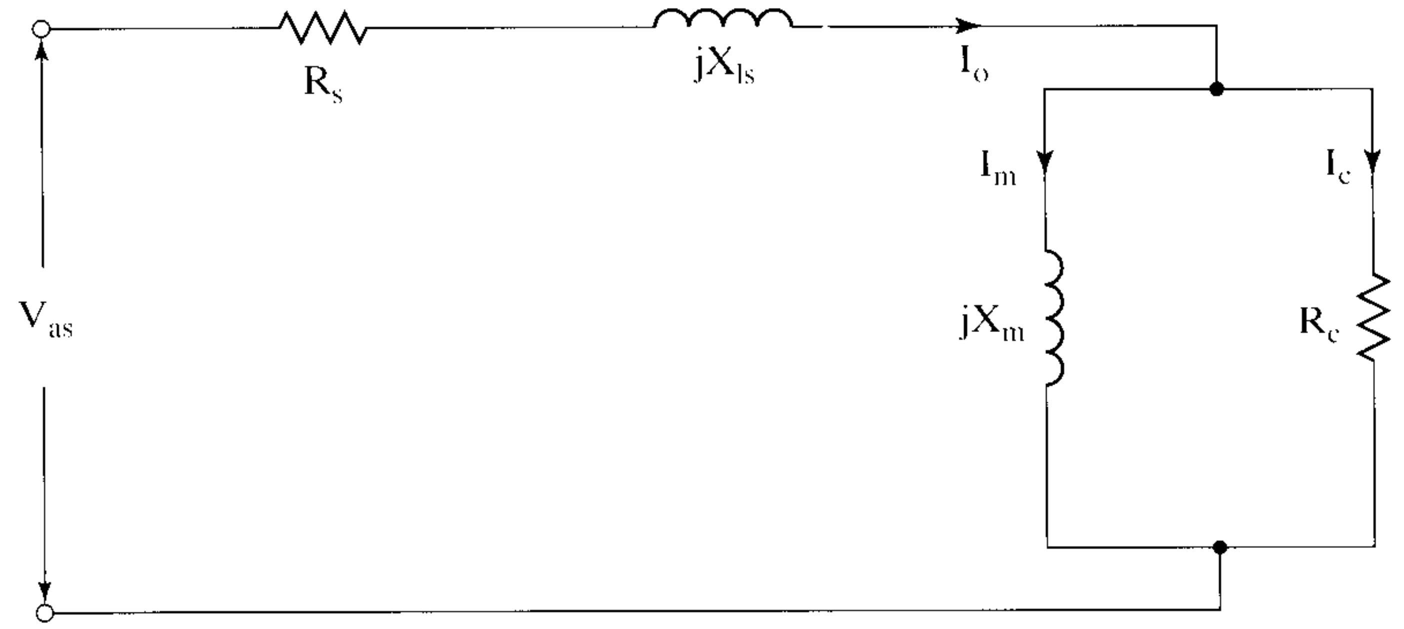

Figure 5.10 Equivalent circuit of the induction motor at no load

The magnetizing current

$$I_m = I_o \sin\phi_o$$

The core-loss current

$$I_c = I_o \cos\phi_o$$

The magnetizing inductance

$$L_m = \frac{V_{as}}{2\pi f_s I_m}$$

The core-loss resistance

$$R_c = \frac{V_{as}}{I_c}$$

- Locked-rotor test

Fig. 5.11 equivalent circuit of the induction motor at standstill.

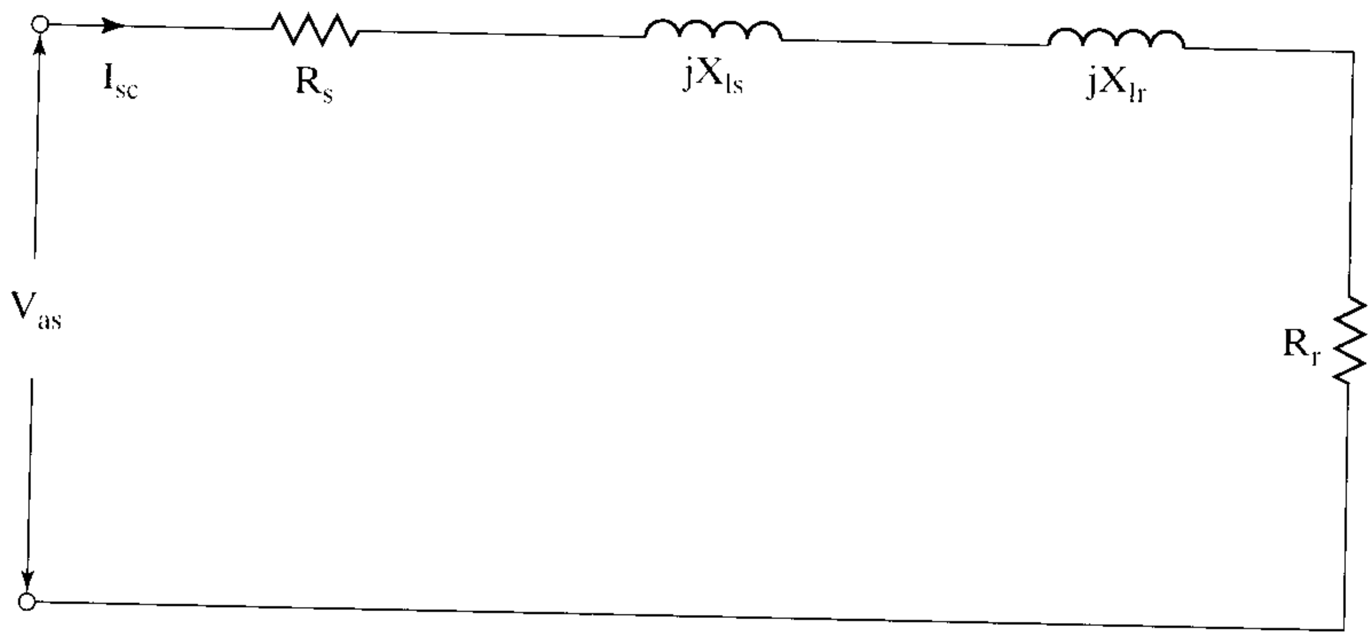

Figure 5.11 Equivalent circuit of the induction motor at standstill

The short-circuit power factor

$$\cos\phi_{sc} = \frac{P_{sc}}{V_{sc}I_{sc}}$$

The short-circuit impedance

$$Z_{sc} = \frac{V_{sc}}{I_{sc}}$$

The rotor resistance and total leakage reactance

$$R_r = Z_{sc}\cos\phi_{sc} - R_s \qquad X_{eq} = Z_{sc}\sin\phi_{sc}$$

The total leakage reactance (the sum of the stator and referred-rotor leakage reactance)

$$X_{eq} = X_{1s} + X_{1r}$$

5.7 Dynamic modeling of induction machine

- Fig. 5.12 Dynamic modeling of the three-phase induction motor.

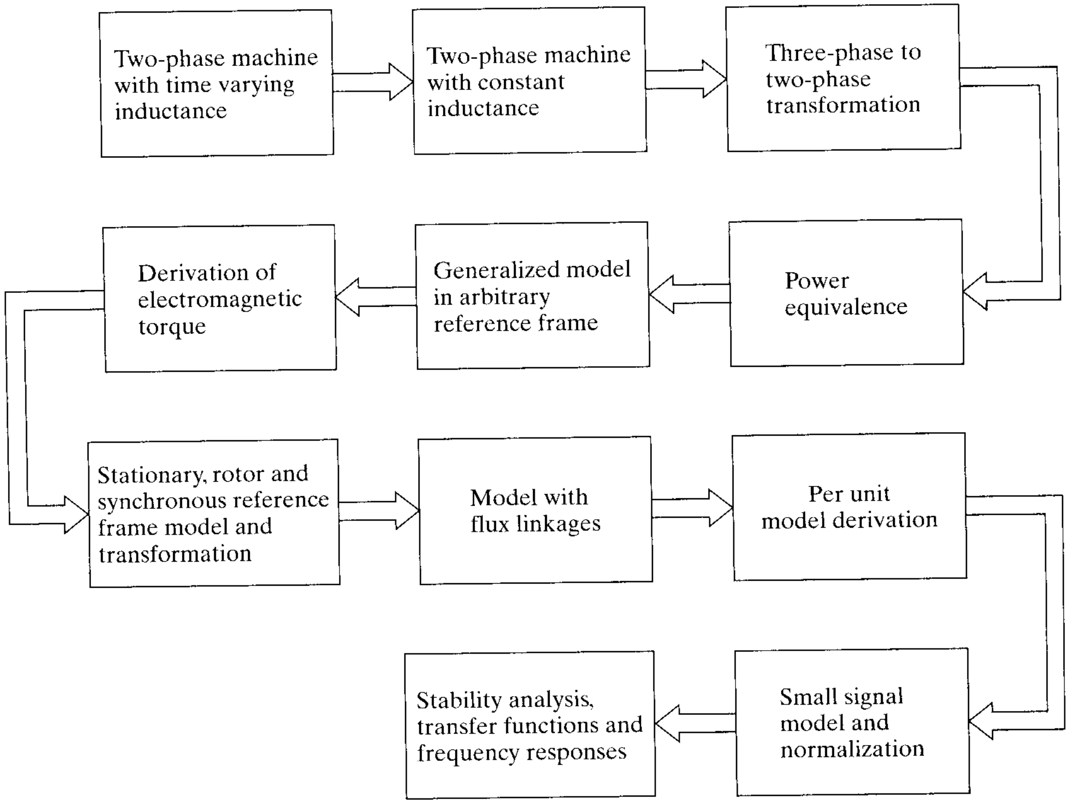

Figure 5.12 Dynamic modeling of the three-phase induction motor

5.7.1 Real-time model of a two-phase induction machine

- Assumptions:

(i) uniform air gap;

(ii) balanced rotor and stator windings, with sinusoidally distributed mmf;

(iii) inductance vs. rotor position is sinusoidal;

(iv) saturation and parameter changes are neglected.

- Fig. 5.13 is a two-phase induction machine with stator and rotor windings.

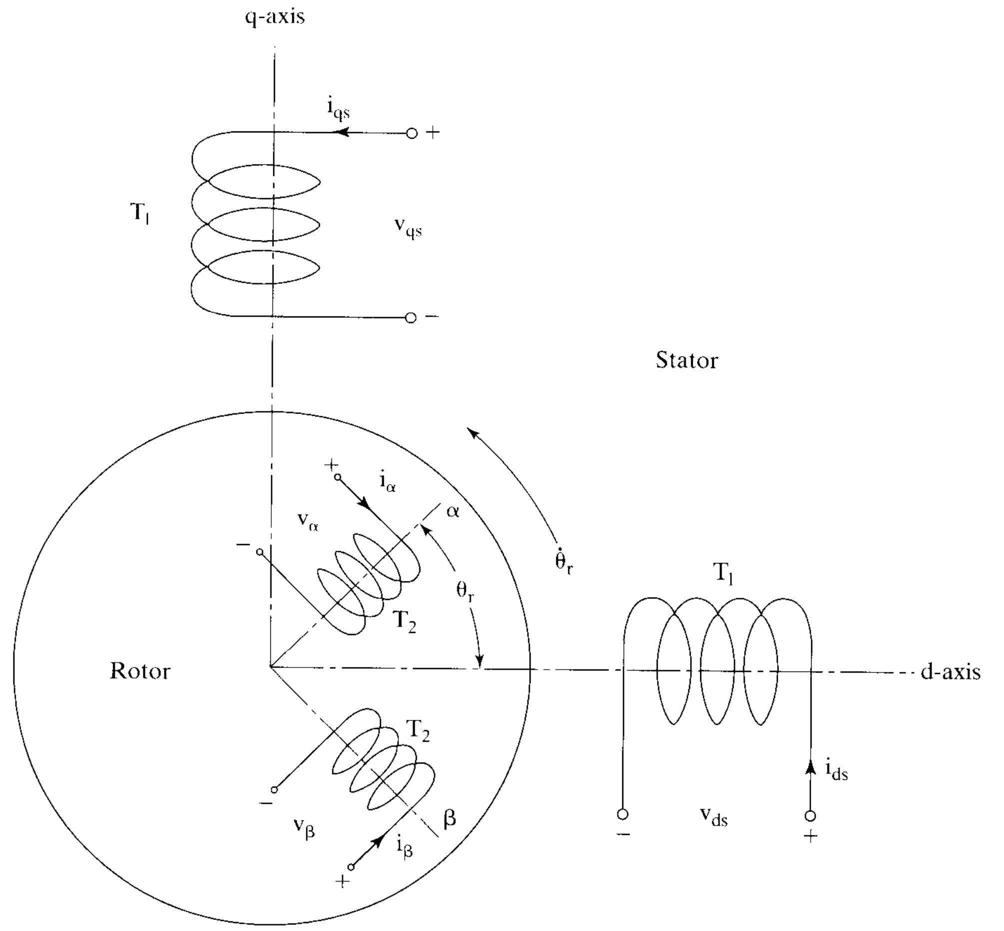

Figure 5.13 Stator and rotor windings of a two-phase induction motor

- The terminal voltage of the stator and rotor windings:

$v_{qs}=R_q i_{qs}+p(L_{qq}i_{qs})+p(L_{qd}i_{ds})+p(L_{q\alpha}i_{\alpha})+p(L_{q\beta}i_{\beta})$

$v_{ds}= p(L_{dq}i_{qs})+R_d i_{ds}+p(L_{dd}i_{ds})+p(L_{d\alpha}i_{\alpha})+p(L_{d\beta}i_{\beta})$

$v_{\alpha}=p(L_{\alpha q}i_{qs})+p(L_{\alpha d}i_{ds})+R_{\alpha}i_{\alpha}+p(L_{\alpha\alpha}i_{\alpha})+p(L_{\alpha\beta}i_{\beta})$

$v_{\beta}=p(L_{\beta q}i_{qs})+p(L_{\beta d}i_{ds})+p(L_{\beta\alpha}i_{\alpha})+ R_{\beta}i_{\beta}+p(L_{\beta\beta}i_{\beta})$

- The self-inductances are independent of angular positions, hence

$L_{\alpha\alpha} = L_{\beta\beta} = L_{rr}; \quad L_{dd} = L_{qq} = L_s.$

- The mutual inductances

$L_{\alpha d} = L_{d\alpha} = L_{sr}\cos\theta_r$; $L_{\beta d} = L_{d\beta} = L_{sr}\sin\theta_r$;

$L_{\alpha q} = L_{q\alpha} = L_{sr}\sin\theta_r$; $L_{\beta q} = L_{q\beta} = -L_{sr}\cos\theta_r$;

- The resulting terminal voltage

$v_{qs} = (R_s+L_s p)i_{qs}+L_{sr}p(i_\alpha \sin\theta_r)-L_{sr}p(i_\beta \cos\theta_r)$

$v_{ds} = (R_s+L_s p)i_{qs}+L_{sr}p(i_\alpha \cos\theta_r)-L_{sr}p(i_\beta \sin\theta_r)$

$v_\alpha = L_{sr}p(i_{qs}\sin\theta_r)+ L_{sr}p(i_{ds}\cos\theta_r)+(R_{rr}+L_{rr}\, p)i_\alpha$

$v_\beta = -L_{sr}p(i_{qs}\cos\theta_r)+ L_{sr}p(i_{ds}\sin\theta_r)+(R_{rr}+L_{rr}\, p)i_\beta$

where $R_s=R_q=R_d$; $R_{rr}=R_\alpha=R_\beta$.

5.7.2 Transformation to obtain constant matrix

- The relationship between the actual currents and the fictitious rotor currents

$$\begin{bmatrix} i_{drr} \\ i_{qrr} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ \sin\theta_r & -\cos\theta_r \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

- The above eq. can be written compactly as

$i_{dqrr} = [T_{ab}] i_{\alpha\beta}$

where $i_{dqrr} = [i_{drr} \quad i_{qrr}]^t$, $i_{\alpha\beta} = [i_\alpha \quad i_\beta]^t$, and

$$T_{\alpha\beta} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ \sin\theta_r & -\cos\theta_r \end{bmatrix}$$

- Fig. 5.14 shows transformation of actual to fictitious variables

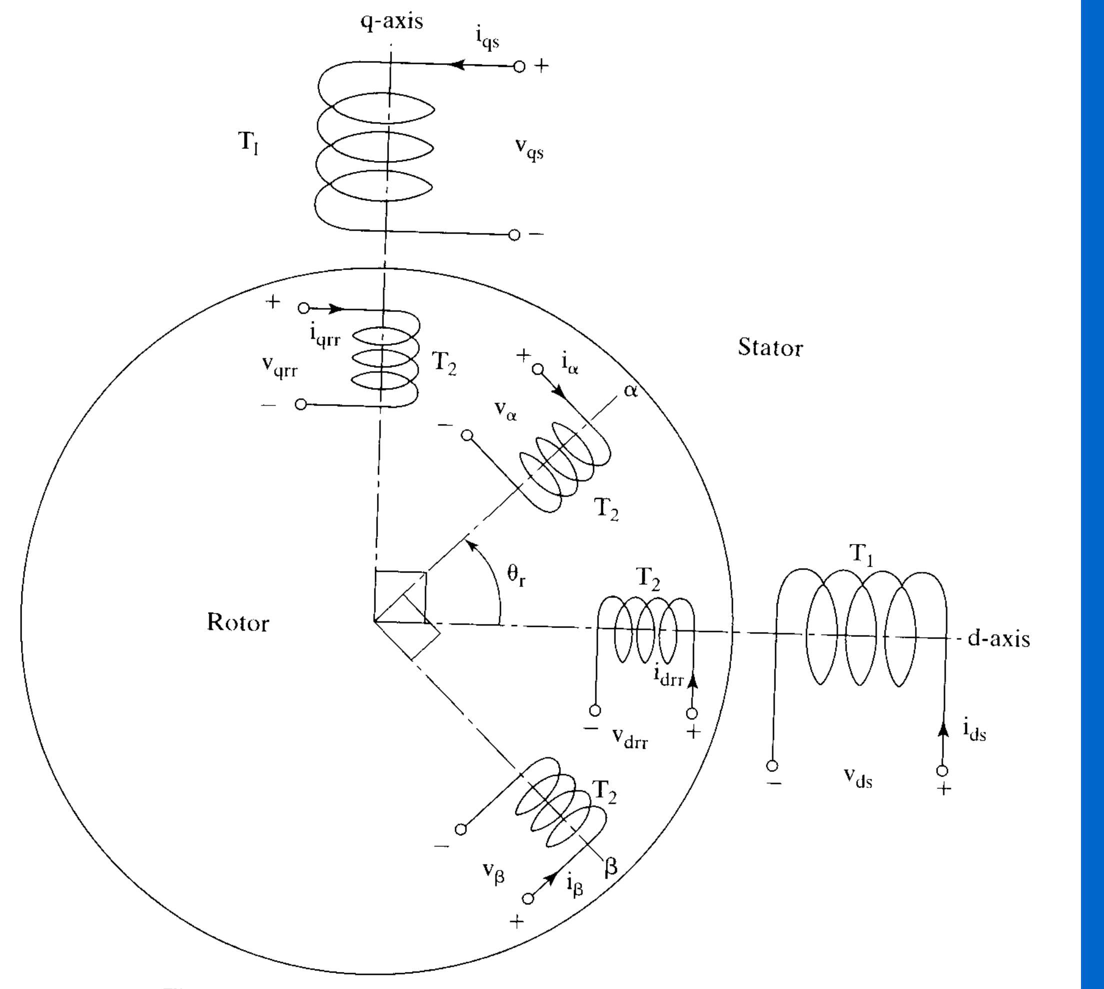

Figure 5.14 Transformation of actual to fictitious rotor variables

- The transformation is valid for voltages, currents, and flux-linkage in a machine.
- The transformation matrix is both orthogonal and symmetric, and satisfies

$$T_{\alpha\beta} = T_{\alpha\beta}^{-1}$$

- We thus have

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{qrr} \\ v_{drr} \end{bmatrix} = \begin{bmatrix} R_s + L_s p & 0 & L_{sr} p & 0 \\ 0 & R_s + L_s p & 0 & L_{sr} p \\ L_{sr} p & -L_{sr}\dot{\theta}_r & R_{rr} + L_{rr} p & -L_{rr}\dot{\theta}_r \\ L_{sr}\dot{\theta}_r & L_{sr} p & L_{rr}\dot{\theta}_r & R_{rr} + L_{rr} p \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qrr} \\ i_{drr} \end{bmatrix}$$

- Let

$$a = \frac{k_{w1}T_1}{k_{w2}T_2}$$

- Then

$R_r = a^2 R_{rr}; \quad L_r = a^2 L_{rr};$

$i_{qr} = i_{qrr}/a; \quad i_{dr} = i_{drr}/a;$

$v_{qr} = a v_{qrr}; \quad v_{dr} = a v_{drr}.$

- The magnetizing inductance of the stator

$L_m = a L_{sr}$

- The machine equation referred to the stator

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{qr} \\ v_{dr} \end{bmatrix} = \begin{bmatrix} R_s + L_s p & 0 & L_m p & 0 \\ 0 & R_s + L_s p & 0 & L_m p \\ L_m p & -L_m \dot{\theta}_r & R_r + L_r p & -L_r \dot{\theta}_r \\ L_m \dot{\theta}_r & L_m p & L_r \dot{\theta}_r & R_r + L_r p \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix}$$

- 5.7.3 Three-phase to two-phase transformation
- The relationship between dqo and abc current

$$\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_o \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta_c & \cos(\theta_c - \frac{2\pi}{3}) & \cos(\theta_c + \frac{2\pi}{3}) \\ \sin\theta_c & \sin(\theta_c - \frac{2\pi}{3}) & \sin(\theta_c + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \Leftrightarrow i_{qdo} = [T_{abc}] i_{abc}$$

- The zero-sequence current, i_o, does not produce a resultant magnetic field.
- The transformation from two-phase currents to three-phase currents can be obtained as

$$i_{abc} = [T_{abc}]^{-1} i_{qdo}$$

where

$$[T_{abc}]^{-1} = \begin{bmatrix} \cos\theta_c & \sin\theta_c & 1 \\ \cos(\theta_c - \frac{2\pi}{3}) & \sin(\theta_c - \frac{2\pi}{3}) & 1 \\ \cos(\theta_c + \frac{2\pi}{3}) & \sin(\theta_c + \frac{2\pi}{3}) & 1 \end{bmatrix}$$

• Fig 5.15 Two- and three-phase stator windings

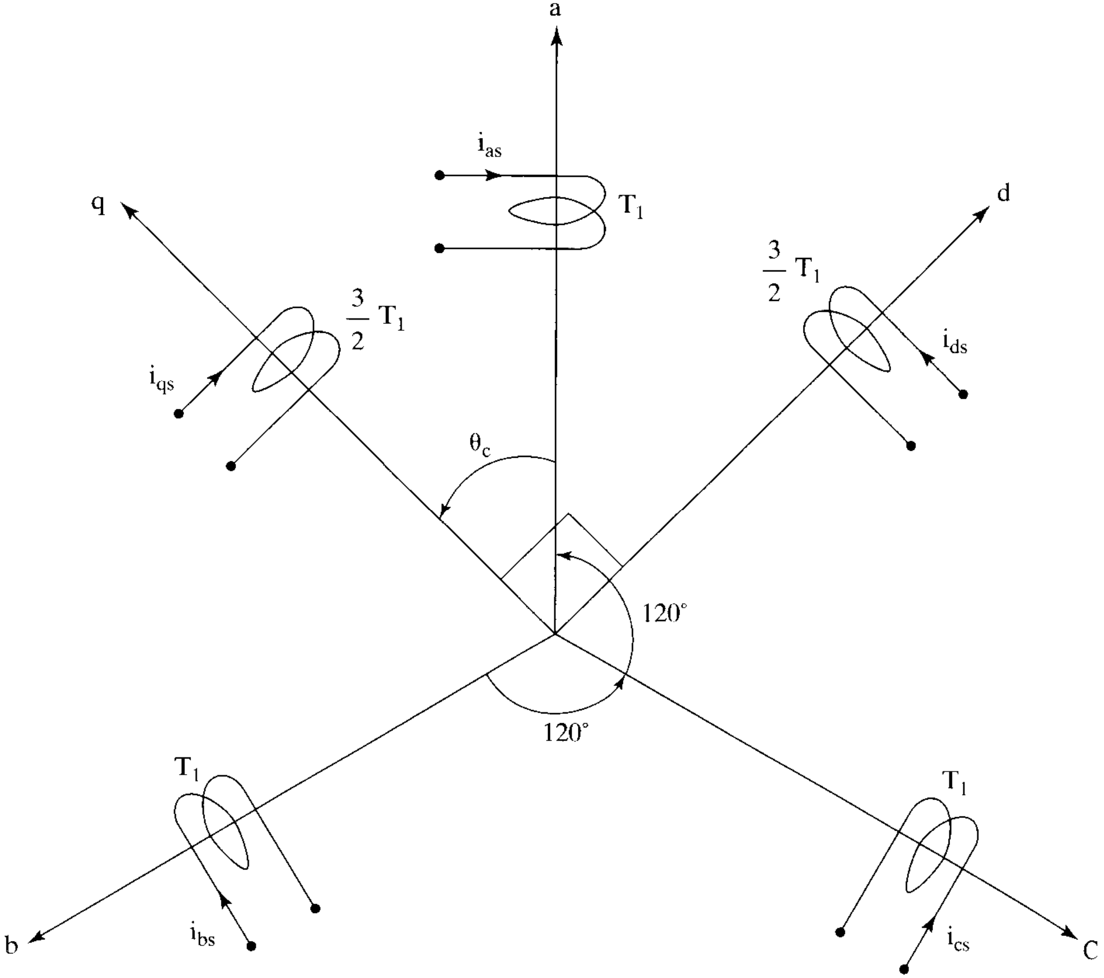

Figure 5.15 Two- and three-phase stator windings

- Under unbalanced condition, two more system equations are

$v_{os} = R_s + L_{1s} p i_{os}$

$v_{or} = R_r + L_{1r} p i_{or}$

- Stanley's model (the stator reference frames model): in that case $\theta_c = 0$, the transformation from abc to dqo variables is

$$T_{abc}^{s} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

- 5.7.4 Power equivalent
- The three-phase instantaneous power input

$$P_i = v_{abc}^t i_{abc} = v_{as} i_{as} + v_{bs} i_{bs} + v_{cs} i_{cs}$$

$$= v_{qdo}^t ([T_{abc}]^{-1})[T_{abc}]^{-1} i_{qdo}$$

$$= \frac{3}{2}((v_{qs} i_{qs} + v_{ds} i_{ds}) + 2 v_o i_o)$$

- 5.7.5 Generalized model in arbitrary reference frames
- Arbitrary reference frame: reference frame rotating at an arbitrary speed.
- The relationships between the currents of reference frame and the arbitrary reference frame are written as

$$i_{qds} = [T^c] i^c_{qds}$$

where $i_{qds} = [i_{qs} \quad i_{ds}]^t$

$$i^c_{qds} = [i^c_{qs} \quad i^c_{dc}]^t$$

$$T^c = \begin{bmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{bmatrix}$$

- Fig. 5.16 shows stationary and arbitrary reference frames.

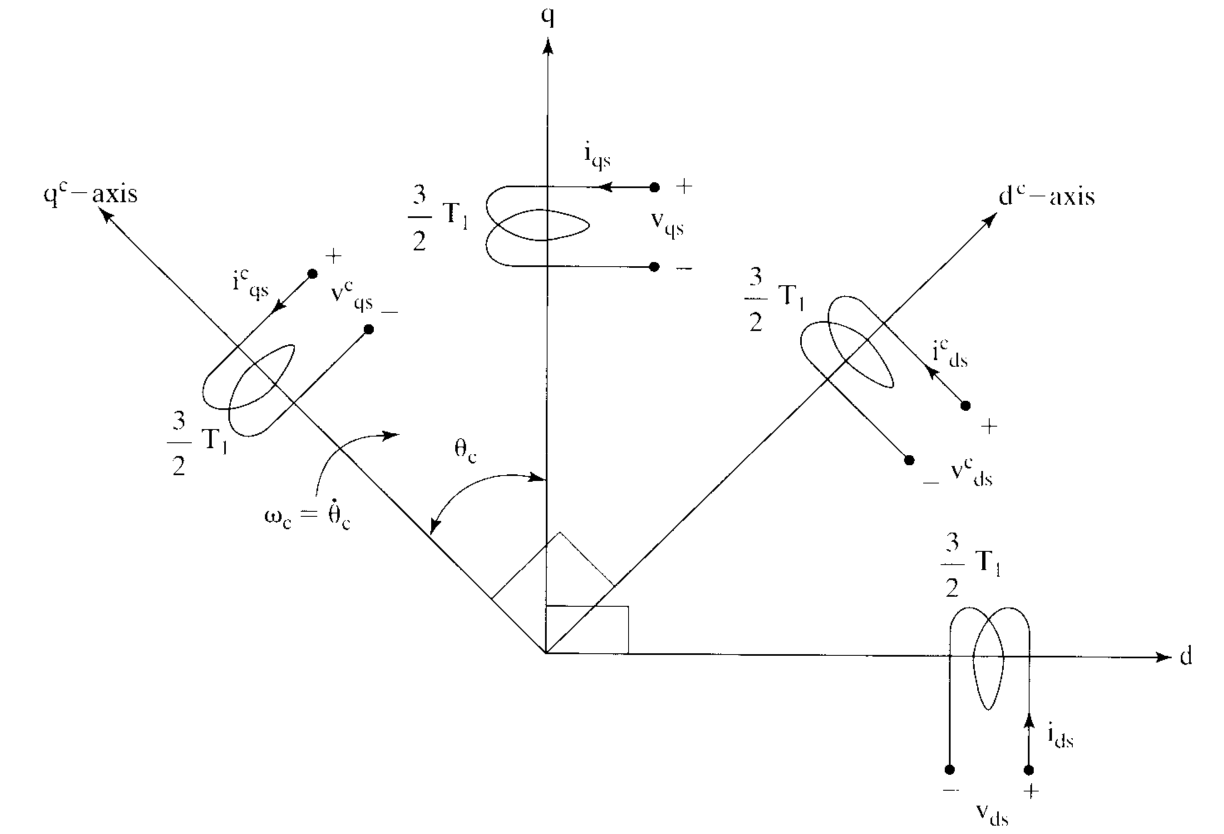

Figure 5.16 Stationary and arbitrary reference frames

- The speed of the arbitrary reference frames is

$$\dot{\theta}_c = \omega_c$$

- Similarly, the fictitious rotor currents transformed into arbitrary frames are

$$i_{qdr} = [T^c] i^c_{qdr}$$

- Likewise, the voltage relationships are

$$v_{qds} = [T^c] v^c_{qds}$$

$$v_{qdr} = [T^c] v^c_{qdr}$$

- The induction-motor model in arbitrary reference frames is obtained as

$$\begin{bmatrix} v_{qs}^{c} \\ v_{ds}^{c} \\ v_{qr}^{c} \\ v_{dr}^{c} \end{bmatrix} = \begin{bmatrix} R_s + L_s p & \omega_c L_s & L_m p & \omega_c L_m \\ -\omega_c L_s & R_s + L_s p & -\omega_c L_m & L_m p \\ L_m p & (\omega_c - \omega_r) L_m & R_r + L_r p & (\omega_c - \omega_r) L_r \\ -(\omega_c - \omega_r) L_m & L_m p & -(\omega_c - \omega_r) L_r & R_r + L_r p \end{bmatrix} \begin{bmatrix} i_{qs}^{c} \\ i_{ds}^{c} \\ i_{qr}^{c} \\ i_{dr}^{c} \end{bmatrix}$$

- The qdo currents in the arbitrary reference frames are obtained as

$$i_{qdo}^{c} = \begin{bmatrix} [T^{c}]^{-1} & 0 \\ 0 & 1 \end{bmatrix} [T_{abc}^{s}][i_{abc}] = [T_{abc}][i_{abc}]$$

- 5.7.6 Electromagnetic torque
- The voltage equation can be written as

$V = [R]i + [L]pi + [G]\omega_r i + [F]\omega_c i$

- The instantaneous input power is

$p_i = i^tV = i^t[R]i + i^t[L]pi + i^t[G]\omega_r i + i^t[F]\omega_c i$

- The air-gap power is derived as

$\omega_m T_e = P_a = i^t[G]i\times\omega_r$

- Substituting for ω_r in terms of ω_m leads as

$T_e = i^t[G]iP/2$

- The electromagnetic torque is obtained as

$$T_e = \frac{3}{2}\frac{P}{2} L_m (i_{qs}^c i_{dr}^c - i_{ds}^c i_{qr}^c)$$

- 5.7.7 Derivation of commonly used induction-motor models

- Stator reference frames model ($\omega_c = 0$)

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{qr} \\ v_{dr} \end{bmatrix} = \begin{bmatrix} R_s + L_s p & 0 & L_m p & 0 \\ 0 & R_s + L_s p & 0 & L_m p \\ L_m p & -\omega_r L_m & R_r + L_r p & -\omega_r L_r \\ \omega_r L_m & L_m p & \omega_r L_r & R_r + L_r p \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix}$$

The torque equation is

$$T_e = \frac{3}{2}\frac{P}{2}L_m(i_{qs}i_{dr} - i_{ds}i_{qr})$$

For a balanced polyphase supply input, the stator q and d axes voltage are

$$v_{qs} = v_{as}$$

$$v_{ds} = \frac{(v_{cs} - v_{bs})}{\sqrt{3}}$$

- Rotor reference frames model ($\omega_c=\omega_r$; $\theta_c=\theta_r$)

$$\begin{bmatrix} v^r_{qs} \\ v^r_{ds} \\ v^r_{qr} \\ v^r_{dr} \end{bmatrix} = \begin{bmatrix} R_s + L_s p & \omega_r L_s & L_m p & \omega_r L_m \\ -\omega_r L_s & R_s + L_s p & -\omega_r L_m & L_m p \\ L_m p & 0 & R_r + L_r p & 0 \\ 0 & L_m p & 0 & R_r + L_r p \end{bmatrix} \begin{bmatrix} i^r_{qs} \\ i^r_{ds} \\ i^r_{qr} \\ i^r_{dr} \end{bmatrix}$$

The electromagnetic torque is

$$T_e = \frac{3}{2}\frac{P}{2}L_m(i^r_{qs}i^r_{dr} - i^r_{ds}i^r_{qr})$$

The transformation from abc to dqo variables

$$[T^r_{abc}] = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \sin\theta_r & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

- Synchronously rotating reference frames model ($\omega_c = \omega_s$; $\theta_c = \theta_s = \omega_s t$)

The model

$$\begin{bmatrix} v_{qs}^e \\ v_{ds}^e \\ v_{qr}^e \\ v_{dr}^e \end{bmatrix} = \begin{bmatrix} R_s + L_s p & \omega_c L_s & L_m p & \omega_c L_m \\ -\omega_c L_s & R_s + L_s p & -\omega_c L_m & L_m p \\ L_m p & (\omega_s - \omega_r) L_m & R_r + L_r p & (\omega_s - \omega_r) L_r \\ -(\omega_s - \omega_r) L_m & L_m p & -(\omega_s - \omega_r) L_r & R_r + L_r p \end{bmatrix} \begin{bmatrix} i_{qs}^e \\ i_{ds}^e \\ i_{qr}^e \\ i_{dr}^e \end{bmatrix}$$

The electromagnetic torque

$$T_e = \frac{3}{2} \frac{P}{2} L_m (i_{qs}^e i_{dr}^e - i_{ds}^e i_{qr}^e)$$

The transformation from abc to dqo variables

$$[T_{abc}^r] = \frac{2}{3} \begin{bmatrix} \cos\theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \sin\theta_r & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

- 5.7.8 Equations in flux linkage
- The stator and rotor flux linkage in the arbitrary reference frames

$$\left.\begin{aligned}\lambda_{qs}^{c} &= L_s i_{qs}^{c} + L_m i_{qr}^{c} \\ \lambda_{ds}^{c} &= L_s i_{ds}^{c} + L_m i_{dr}^{c} \\ \lambda_{qr}^{c} &= L_r i_{qr}^{c} + L_m i_{qs}^{c} \\ \lambda_{dr}^{c} &= L_r i_{dr}^{c} + L_m i_{ds}^{c}\end{aligned}\right\}$$

- The zero-sequence flux linkage are

$$\left.\begin{aligned}\lambda_{os} &= L_{1s} i_{os} \\ \lambda_{or} &= L_{1r} i_{or}\end{aligned}\right\}$$

- The q axis stator voltage in the arbitrary reference frame is

$$v_{qs}^{c} = R_s i_{qs}^{c} + \omega_c (L_s i_{ds}^{c} + L_m i_{dr}^{c}) + L_m p i_{qr}^{c} + L_s p i_{qs}^{c}$$

$$v_{qs}^{c} = R_s i_{qs}^{c} + \omega_c \lambda_{ds}^{c} + p\lambda_{qs}^{c}$$

- Similarly,

$$v_{ds}^{c} = R_s i_{ds}^{c} - \omega_c \lambda_{qs}^{c} + p\lambda_{ds}^{c}$$

$$v_{os}^{c} = R_s i_{os} + p\lambda_{os}$$

$$v_{qr}^{c} = R_r i_{dr}^{c} + (\omega_c - \omega_r)\lambda_{dr}^{c} + p\lambda_{qr}^{c}$$

$$v_{dr}^{c} = R_r i_{dr}^{c} - (\omega_c - \omega_r)\lambda_{qr}^{c} + p\lambda_{dr}^{c}$$

$$v_{or}^{c} = R_r i_{or} + p\lambda_{or}$$

- For normalization of the variables, a modified flux linkage is defined as

$$\psi^c_{qs} = \omega_b \lambda^c_{qs} = \omega_b (L_s i^c_{qs} + L_m i^c_{qr}) = X_s i^c_{qs} + X_m i^c_{qr}$$

- The other modified flux linkage are

$$\psi^c_{qs} = X_s i^c_{ds} + X_m i^c_{dr}$$

$$\psi^c_{qr} = X_r i^c_{qs} + X_m i^c_{qs}$$

$$\psi^c_{dr} = X_r i^c_{dr} + X_m i^c_{ds}$$

$$\psi_{os} = X_{1s} i_{os}$$

$$\psi_{or} = X_{1r} i_{or}$$

- The flux linkage in modified terms

$$\lambda^c_{qs} = \frac{\psi^c_{qs}}{\omega_b}, \lambda^c_{ds} = \frac{\psi^c_{ds}}{\omega_b}, \lambda^c_{os} = \frac{\psi^c_{os}}{\omega_b}, \lambda^c_{qr} = \frac{\psi^c_{qr}}{\omega_b}, \lambda^c_{dr} = \frac{\psi^c_{dr}}{\omega_b}, \lambda^c_{or} = \frac{\psi^c_{or}}{\omega_b},$$

- The resulting equations in modified flux linkage

$$v_{qs}^{c} = R_s i_{qs}^{c} + \frac{\omega_c}{\omega_b}\psi_{ds}^{c} + \frac{p}{\omega_b}\psi_{qs}^{c}$$

$$v_{ds}^{c} = R_s i_{ds}^{c} - \frac{\omega_c}{\omega_b}\psi_{qs}^{c} + \frac{p}{\omega_b}\psi_{ds}^{c}$$

$$v_{os} = R_s i_{os} + \frac{p}{\omega_b}\psi_{os}$$

$$v_{qr}^{c} = R_r i_{qr}^{c} + \frac{(\omega_c - \omega_r)}{\omega_b}\psi_{dr}^{c} + \frac{p}{\omega_b}\psi_{qr}^{c}$$

$$v_{dr}^{c} = R_r i_{dr}^{c} - \frac{(\omega_c - \omega_r)}{\omega_b}\psi_{qr}^{c} + \frac{p}{\omega_b}\psi_{dr}^{c}$$

$$v_{or} = R_r i_{or} + \frac{p}{\omega_b}\psi_{or}$$

- The electromagnetic torque in flux linkage and currents is

$$T_e = \frac{3}{2}\frac{P}{2} L_m (i_{qs}^c i_{dr}^c - i_{ds}^c i_{qr}^c) = \frac{3}{2}\frac{P}{2} (i_{qs}^c (L_m i_{dr}^c) - i_{ds}^c (L_m i_{qr}^c))$$

$$= \frac{3}{2}\frac{P}{2} (i_{qs}^c (\lambda_{ds}^c - L_s i_{ds}^c) - i_{ds}^c (\lambda_{qs}^c - L_s i_{qs}^c)) = \frac{3}{2}\frac{P}{2} (i_{qs}^c \lambda_{ds}^c - i_{ds}^c \lambda_{qs}^c)$$

- Alternatively, the electromagnetic torque in terms of modified flux linkage and currents

$$T_e = \frac{3}{2}\frac{P}{2}\frac{1}{\omega_b} (i_{qs}^c \psi_{ds}^c - i_{ds}^c \psi_{qs}^c)$$

- # 5.7.9 Per-Unit model
- Base Power = $P_b = 3V_{b3}I_{b3}$
- The base quantities in dq frames

$$V_b = \sqrt{2}V_{b3} \quad I_b = \sqrt{2}I_{b3}$$

- Hence, the base power is defined as

$$P_b = 3V_{b3}I_{b3} = 3\frac{V_b}{\sqrt{2}}\frac{I_b}{\sqrt{2}} = \frac{3}{2}V_bI_b$$

- The normalized voltage (Let $V_b = I_bZ_b$)

$$v_{qsn}^c = \frac{v_{qs}^c}{V_b} = \frac{R_s}{V_b}i_{qs}^c + \frac{X_s}{\omega_b V_b}pi_{qs}^c + \frac{\omega_c X_s}{\omega_b V_b}i_{ds}^c + \frac{X_m}{\omega_b V_b}i_{qr}^c + \frac{\omega_c X_m}{\omega_b V_b}i_{dr}^c$$

$$= \frac{R_s}{Z_b}\frac{i_{qs}^c}{I_b} + \frac{1}{\omega_b}\frac{X_s}{Z_b}p(\frac{i_{qs}^c}{I_b}) + \frac{\omega_c}{\omega_b}\frac{X_s}{Z_b}\frac{i_{ds}^c}{I_b} + \frac{1}{\omega_b}\frac{X_m}{Z_b}p(\frac{i_{qr}^c}{I_b}) + \frac{\omega_c}{\omega_b}\frac{X_m}{Z_b}\frac{i_{dr}^c}{I_b}$$

- Similarly, we can get

$$\begin{bmatrix} v_{qsn}^c \\ v_{dsn}^c \\ v_{qrn}^c \\ v_{drn}^c \end{bmatrix} = \begin{bmatrix} R_{sn} + \frac{X_{sn}}{\omega_b} p & \omega_{cn} X_{sn} & \frac{X_{mn}}{\omega_b} p & \omega_{cn} X_{mn} \\ -\omega_{cn} X_{sn} & R_{sn} + \frac{X_{sn}}{\omega_b} p & -\omega_{cn} X_{mn} & \frac{L_{mn}}{\omega_b} p \\ \frac{X_{mn}}{\omega_b} p & (\omega_{cn} - \omega_{rn}) X_{mn} & R_{rn} + \frac{X_{rn}}{\omega_b} p & (\omega_{cn} - \omega_{rn}) X_r \\ -(\omega_{cn} - \omega_{rn}) X_{mn} & \frac{X_{mn}}{\omega_b} p & -(\omega_{cn} - \omega_{rn}) X_{rn} & R_{rn} + \frac{X_{rn}}{\omega_b} p \end{bmatrix} \begin{bmatrix} i_{qsn}^c \\ i_{dsn}^c \\ i_{qrn}^c \\ i_{drn}^c \end{bmatrix}$$

- The normalized electromagnetic torque

$$T_{en} = \frac{T_e}{T_b} = \frac{\frac{3}{2}\frac{P}{2}\frac{1}{\omega_b}}{\frac{P}{2}\frac{P_b}{\omega_b}} (i_{qs}^c \psi_{ds}^c - i_{ds}^c \psi_{qs}^c) = (i_{qsn}^c \psi_{dsn}^c - i_{dsn}^c \psi_{qsn}^c) \quad \text{p.u.}$$

$$T_{en} = 2Hp\omega_{rn} + T_{1n} + B_n \omega_{rn} \quad \text{p.u.} \qquad H = \frac{1}{2}\frac{J\omega_b^2}{P_b (P/2)^2} \qquad B_n = \frac{B\omega_b^2}{P_b (P/2)^2}$$

5.8 Dynamic simulation

- The equations of the induction machine in p.u. are cast in the state-space form as

$$P_1 pX_1 + Q_1 X_1 = u_1$$

where $X_1 = [i^c_{qsn} \quad i^c_{dsn} \quad i^c_{qrn} \quad i^c_{drn}]^t \quad u_1 = [v^c_{qsn} \quad v^c_{dsn} \quad v^c_{qrn} \quad v^c_{drn}]^t$

$$P_1 = \begin{bmatrix} \frac{X_{sn}}{\omega_b} & 0 & \frac{X_{mn}}{\omega_b} & 0 \\ 0 & \frac{X_{sn}}{\omega_b} & 0 & \frac{X_{mn}}{\omega_b} \\ \frac{X_{mn}}{\omega_b} & 0 & \frac{X_{rn}}{\omega_b} & 0 \\ o & \frac{X_{mn}}{\omega_b} & 0 & \frac{X_{rn}}{\omega_b} \end{bmatrix} \quad Q_1 = \begin{bmatrix} R_{sn} & \omega_{cn} X_{mn} & 0 & \omega_{cn} X_{mn} \\ -\omega_{cn} X_{sn} & R_{sn} & -\omega_{cn} X_{sn} & 0 \\ 0 & (\omega_{cn} - \omega_{rn}) X_{mn} & R_{rn} & (\omega_{cn} - \omega_{rn}) X_{rn} \\ -(\omega_{cn} - \omega_{rn}) X_{mn} & 0 & -(\omega_{cn} - \omega_{rn}) X_{rn} & R_{rn} \end{bmatrix}$$

- The above eq. Can be arranged in the state-space form as follows:

$pX_1 = P^{-1}(u_1 - Q_1X_1)$

- This can be written as

$pX_1 = A_1X_1 + B_1u_1$

where $A_1 = -P_1^{-1}Q_1$ and $B_1 = P_1^{-1}$

- The electromechanical torque is

$T_{en} = 2Hpw_{rn} + T_{ln} + B_n\omega_{rn}$

where $T_{en} = (i^c_{qsn}\psi^c_{drn} - i^c_{dsn}\psi^c_{qsn})$ $H = \frac{1}{2}\frac{J\omega_b^2}{P_b(P/2)^2}$

- A convenient form of the electromechanical equation

$$T_{en} = X_{mn}(i^{c}_{qsn} i^{c}_{drn} - i^{c}_{dsn} i^{c}_{qsn})$$

- Hence, the electromechanical equation

$$p\omega_{rn} = \frac{X_{mn}}{2H}(i^{c}_{qsn} i^{c}_{drn} - i^{c}_{dsn} i^{c}_{qrn}) - \frac{T_{ln}}{2H} - \frac{B_{n}}{2H}\omega_{rn}$$

5.9 Small-signal equations of …

- Linearizing the nonlinear dynamic equation around an operating point by using perturbation techniques.
- The variables in SI units after perturbation

$$v_{qs}^{e} = v_{qso}^{e} + \delta v_{qs}^{e}$$
$$v_{ds}^{e} = v_{dso}^{e} + \delta v_{ds}^{e}$$
$$i_{qs}^{e} = i_{qso}^{e} + \delta i_{qs}^{e}$$

- The state-space form

$$pX = AX + B_1U$$

where $X = [\delta i_{qs}^e \quad \delta i_{ds}^e \quad \delta i_{qr}^e \quad \delta i_{dr}^e \quad \delta\omega_r]^t$

$$U = [\delta v_{qs}^e \quad \delta v_{ds}^e \quad \delta v_{qr}^e \quad \delta v_{dr}^e \quad \delta\omega_s \quad \delta T_1]^t$$

$$A = P_1^{-1}Q_1 \qquad B_1 = P_1^{-1}R_1$$

$$P_1 = \begin{bmatrix} L_s & 0 & L_m & 0 & 0 \\ 0 & L_s & 0 & L_m & 0 \\ L_m & 0 & L_r & 0 & 0 \\ 0 & L_m & 0 & L_r & 0 \\ 0 & 0 & 0 & 0 & J \end{bmatrix}$$

- (Continued)

$$Q_1 = \begin{bmatrix} -R_s & -\omega_{so}L_s & 0 & -\omega_{so}L_m & 0 \\ \omega_{so}L_s & -R_s & \omega_{so}L_m & 0 & 0 \\ 0 & -(\omega_{so}-\omega_{ro})L_m & -R_r & -(\omega_{so}-\omega_{ro})L_r & L_m i_{dso}^e + L_r i_{dro}^e \\ (\omega_{so}-\omega_{ro})L_m & 0 & (\omega_{so}-\omega_{ro})L_r & R_r & -(L_m i_{qso}^e + L_r i_{qro}^e) \\ k_2 i_{dro}^e & -k_2 i_{qro}^e & -k_2 i_{dso}^e & k_2 i_{qso}^e & -B \end{bmatrix}$$

$$k_2 = \frac{3}{2}\left(\frac{P}{2}\right)^2 L_m$$

$$R_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & -L_s i_{dso}^e + L_m i_{dro}^e & 0 \\ 0 & 1 & 0 & 0 & -L_s i_{qso}^e + L_m i_{qro}^e & 0 \\ 0 & 0 & 1 & 0 & -L_m i_{dso}^e + L_r i_{dro}^e & 0 \\ 0 & 0 & 0 & 1 & -L_s i_{qso}^e + L_m i_{qro}^e & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{P}{2} \end{bmatrix}$$

5.10 Evaluation of control characteristics ...

- Assumption of zero initial conditions

  $sX(s) = AX(s) + B_1u(s)$

  $y(s) = CX(s) + Du(s)$

- The output can be expressed as

  $y(s) = (C(sI - A)^{-1}B_1 + D)u(s)$

5.11 Space-phasor model

- 5.11.2 DQ flux-linkages model derivation
- The currents in terms of the flux linkage

$$\left.\begin{aligned}
i_{qs}^{c} &= \frac{1}{\Delta_1}(L_r\lambda_{qs}^{c} - L_m\lambda_{qr}^{c}) \\
i_{ds}^{c} &= \frac{1}{\Delta_1}(L_r\lambda_{ds}^{c} - L_m\lambda_{dr}^{c}) \\
i_{qr}^{c} &= \frac{1}{\Delta_1}(-L_m\lambda_{qs}^{c} + L_s\lambda_{qr}^{c}) \\
i_{dr}^{c} &= \frac{1}{\Delta_1}(-L_m\lambda_{ds}^{c} + L_s\lambda_{dr}^{c})
\end{aligned}\right\}$$

where $\Delta_1 = (L_s L_r - L_m^2)$

- The model in arbitrary reference frames in normalized units is derived as

$$\frac{d}{d\tau}\begin{bmatrix}\lambda_{qsn}^{c}\\ \lambda_{dsn}^{c}\\ \lambda_{qm}^{c}\\ \lambda_{drn}^{c}\end{bmatrix}=\begin{bmatrix}-\frac{1}{\tau_s'} & -\omega_{cn} & \frac{k_r}{\tau_s'} & 0\\ \omega_{cn} & -\frac{1}{\tau_s'} & 0 & \frac{k_r}{\tau_s'}\\ \frac{k_r}{\tau_s'} & 0 & -\frac{1}{\tau_s'} & \omega_{cn}\\ 0 & \frac{k_r}{\tau_s'} & \omega_{cn} & \end{bmatrix}\begin{bmatrix}\lambda_{qsn}^{c}\\ \lambda_{dsn}^{c}\\ \lambda_{qm}^{c}\\ \lambda_{drn}^{c}\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}v_{qsn}^{c}\\ v_{dsn}^{c}\end{bmatrix}$$

- Fig. 5.20 Signal-flow graph.

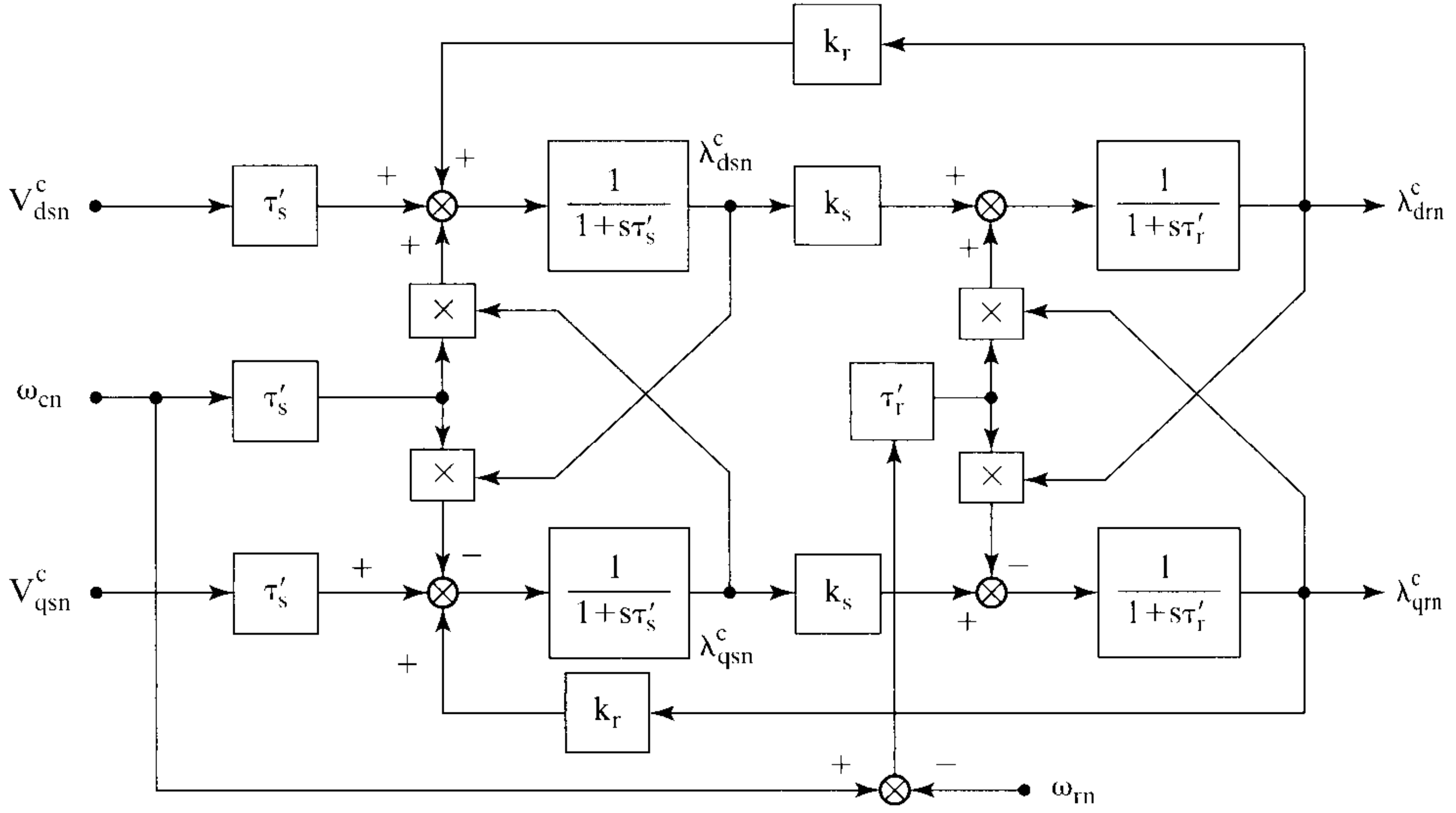

Figure 5.20 Signal-flow graph of the induction machine

• Fig. 5.21 Root loci (in stator reference frames with rotor speed varying 0 ~ 0.5 p.u.

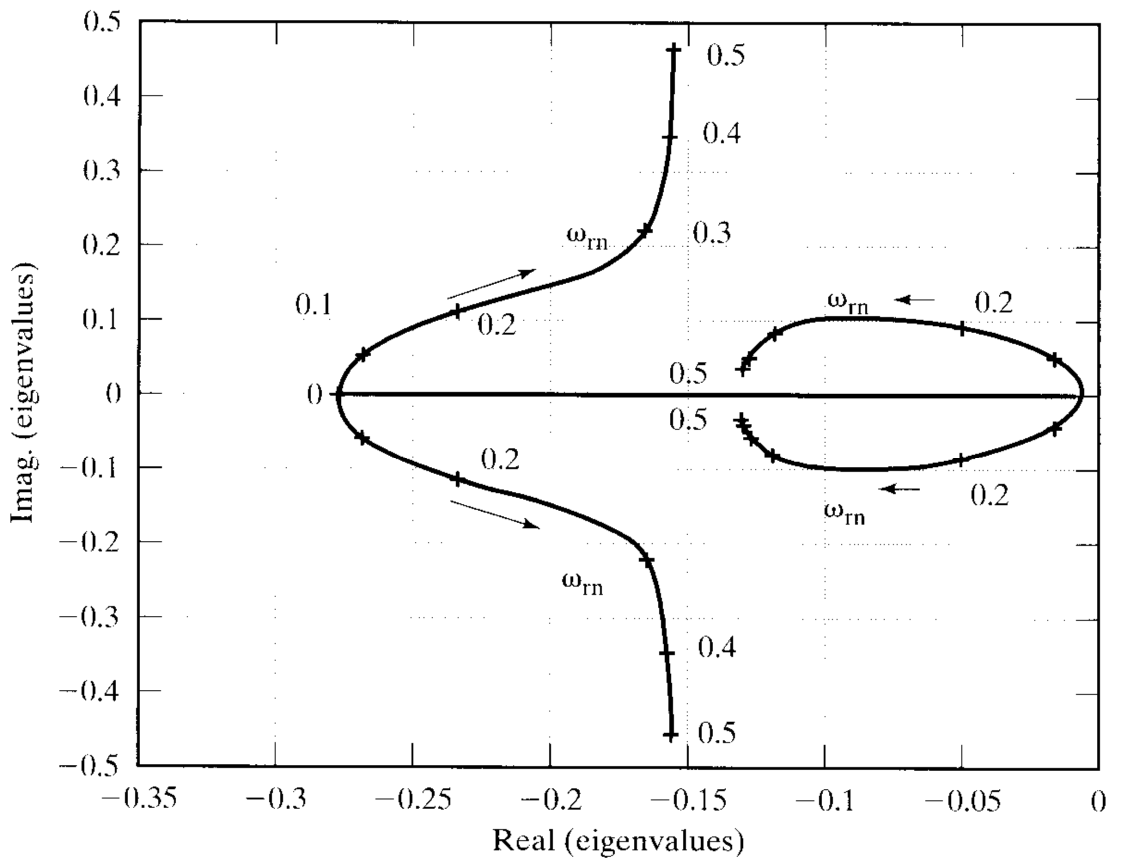

Figure 5.21 Loci of the eigenvalues in stator reference frames with rotor speed varying from 0 to 0.5 p.u.

- Fig. 5.22 Root loci (in arbitrary reference frames with velocity of 0.5 p.u. and rotor speed varying from 0 to 0.5 p.u.)

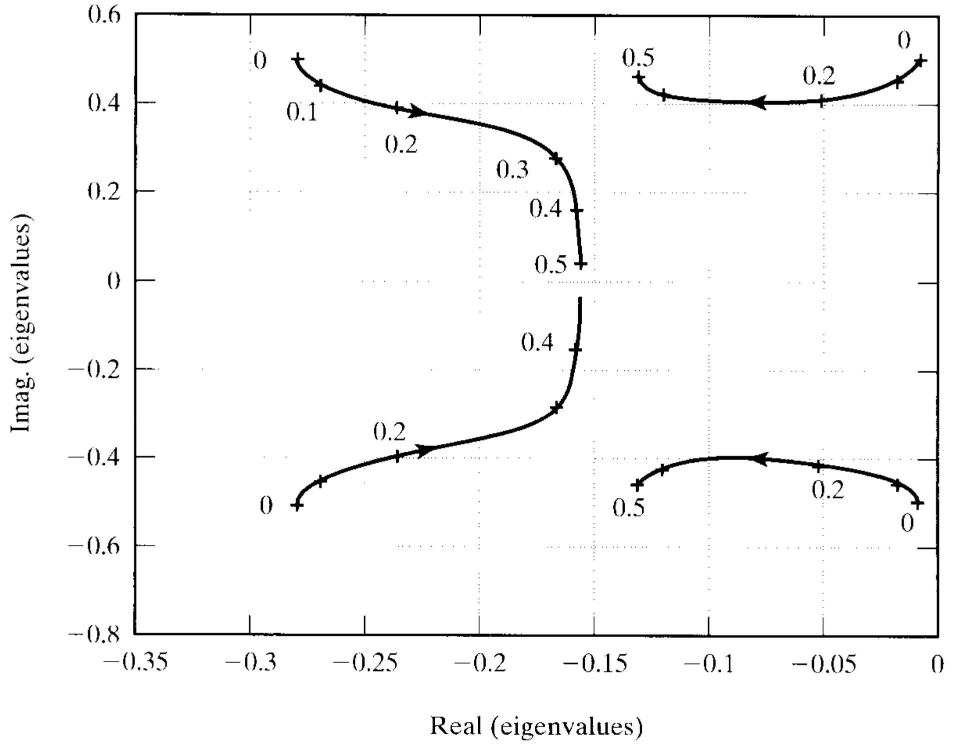

Figure 5.22 Loci of the eigenvalues in arbitrary reference frames with a velocity of 0.5 p.u. and rotor speed varying from 0 to 0.5 p.u.

5.11.4 Space-Phasor model derivation

- Let the space phasor of the stator flux linkage in arbitrary reference frames be

$$\lambda_{sn}^{c} = \lambda_{qsn}^{c} - j\lambda_{dsn}^{c}$$

- A complex phasor form

$$\lambda_{sn}^{c} = e^{-j\omega_{cn}t}\{\frac{2}{3}(\lambda_{asn} + e^{j2\pi/2}\lambda_{bsn} + e^{j4\pi/3}\lambda_{csn})\} = e^{-j\omega_{cn}t}(\lambda_{qsn} - j\lambda_{dsn})$$

- Applying the space-phasor definition to the expression given in (5.265)

$$\frac{d\lambda_{sn}^{c}}{d\tau} + (\frac{1}{\tau_s'} + j\omega_{cn})\lambda_{sn}^{c} = \frac{k_r}{\tau_s'}\lambda_{rn}^{c} + v_{sn}^{c}$$

$$\frac{d\lambda_{rn}^{c}}{d\tau} + [\frac{1}{\tau_r'} + j(\omega_{cn} - \omega_{rn})]\lambda_{rn}^{c} = \frac{k_r}{\tau_r'}\lambda_{sn}^{c}$$

- Fig. 5.23 Signal-flow graph of the space-phasor-mpdeled induction machine.

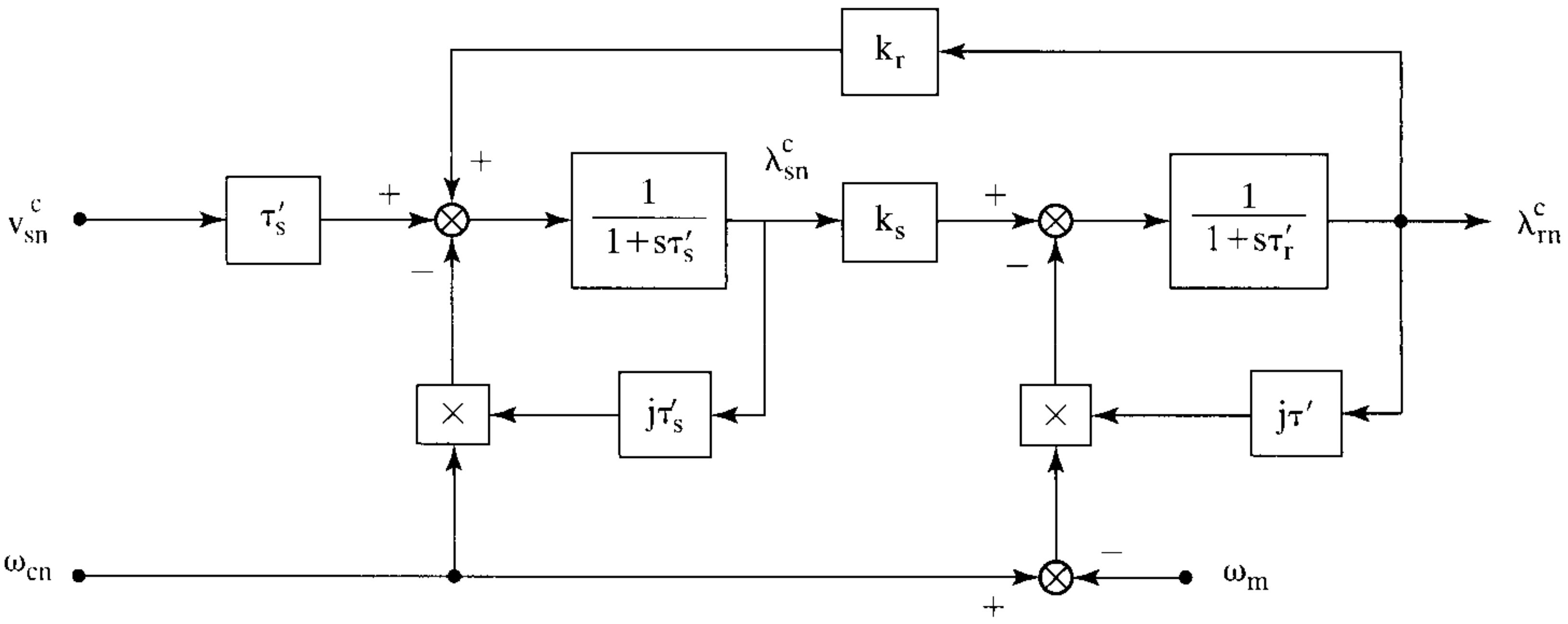

Figure 5.23 Signal-flow graph of the space-phasor-modeled induction machine

• Fig. 5.24 Root loci (in stator reference frames with the rotor speed varied 0~0.5 p.u.

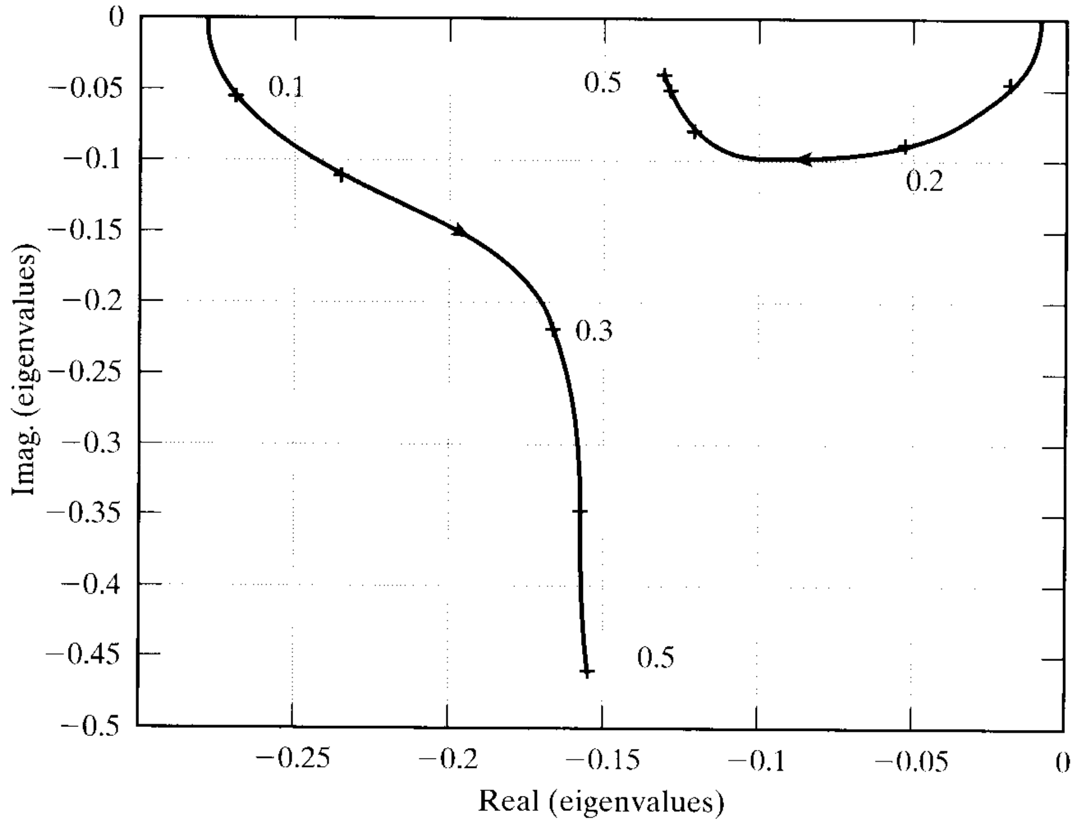

Figure 5.24 Loci of the eigenvalues in stator reference frames, with the rotor speed varied from 0 to 0.5 p.u. in a space-phasor model

5.12 Control principle of the induction motor

- The stator current and flux-linkage are defined in q and d axes, respectively

$$i_s = i_{qs}^e - j i_{ds}^e$$

$$\lambda_s = \lambda_{qs}^e - j \lambda_{ds}^e$$

- The input power for balanced supply voltage is given by $p_i = \frac{3}{2}(v_{qs}^e i_{qs}^e + v_{ds}^e i_{ds}^e)$

$$p_i = \frac{3}{2}(R_s[(i_{qs}^e)^2 + (i_{ds}^e)^2] + \omega_s L_m [i_{qs}^e i_{dr}^e - i_{ds}^e i_{qr}^e]$$

$$+ i_{qs}^e [L_s p i_{qs}^e + L_m p i_{qr}^e] + i_{ds}^e [L_s p i_{ds}^e + L_m p i_{dr}^e])$$

- The electromagnetic torque

$$T_e = \frac{3}{2}\frac{P}{2}L_m(\lambda_{ds}^e i_{qs}^e - \lambda_{qs}^e i_{ds}^e) = \frac{3}{2}\frac{P}{2}L_m(i_{qs}^e i_{dr}^e - i_{ds}^e i_{qr}^e)$$

where $\lambda_{qs}^e = L_s i_{qs}^e + L_m i_{qr}^e$

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e$$

- The power input in term of the torque and flux linkage and current:

$$p_i = \frac{3}{2}R_s[(i_{qs}^e)^2 + (i_{ds}^e)^2] + \frac{2}{P}\omega_s T_e + \frac{3}{2}[i_{qs}^e p\lambda_{qs}^e - i_{ds}^e p\lambda_{ds}^e]$$

$$p_i = \frac{3}{2}R_s i^2 + \frac{2}{P}\omega_s \,\mathrm{Im}[i_s \overline{\lambda}_s^e] + \frac{3}{2}\mathrm{Re}[i_s p\overline{\lambda}_s^e]$$

- There are three distinct components of the input power:

First: the stator resistance losses;

Second: the sum of the slip and mechanical power; the shaft power and friction and windage losses.

Third: the rate of change of magnetic energy.